Spring_#8

Heat Transfer

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Ch7. External Forced Convection: Objectives

- Distinguish between internal and external flow.
- Develop an intuitive understanding of (friction drag and pressure drag), and evaluate the average drag and convection coefficients in external flow.
- Evaluate the drag and heat transfer associated with flow over a flat plate for both laminar and turbulent flow.
- Calculate the drag force exerted on cylinders during cross flow, and the average heat transfer coefficient..

DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

- Fluid flow over solid bodies frequently occurs in practice such as the *drag force* acting on the automobiles, power lines, trees, and underwater pipelines; the *lift* developed by airplane wings; *upward draft* of rain, snow, hail, and dust particles in high winds; and the *cooling* of metal or plastic sheets, steam and hot water pipes, and extruded wires.
- **Free-stream velocity:** The velocity of the fluid relative to an immersed solid body sufficiently far from the body.
- It is usually taken to be equal to the **upstream velocity** *V* (**approach velocity**), which is the velocity of the approaching fluid far ahead of the body.
- The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface.

FIGURE 7–1
Flow over bodies is commonly encountered in practice.

DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

The drag force F_D depends on the density ρ of the fluid, the upstream velocity V, and the size, shape, and orientation of the body, among other things.

The drag characteristics of a body is represented by the dimensionless drag coefficient C_D defined as

Drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

The part of drag that is due directly to wall shear stress τ_w is called the skin friction drag (or just *friction drag*) since it is caused by frictional effects, and the part that is due directly to pressure P is called the pressure drag.

$$C_D = C_{D,\,\text{friction}} + C_{D,\,\text{pressure}}$$

Flat plate:

$$C_D = C_{D,\,\text{friction}} = C_f$$

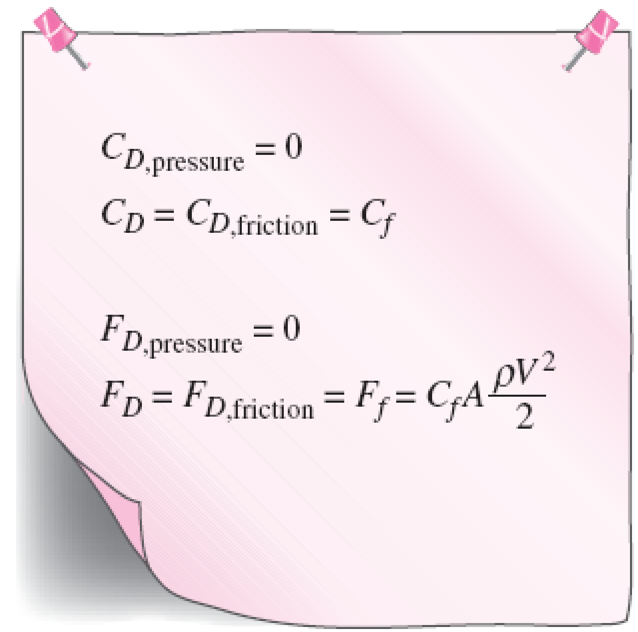


FIGURE 7–4
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

- At low Reynolds numbers, most drag is due to friction drag.
- The friction drag is proportional to the surface area.
- The pressure drag is proportional to the frontal area and to the *difference* between the pressures acting on the front and back of the immersed body.
- The pressure drag is usually dominant for blunt bodies and negligible for streamlined bodies.
- When a fluid separates from a body, it forms a separated region between the body and the fluid stream.
- **Separated region:** The low-pressure region behind the body where recirculating and backflows occur.
- The larger the separated region, the larger the pressure drag.

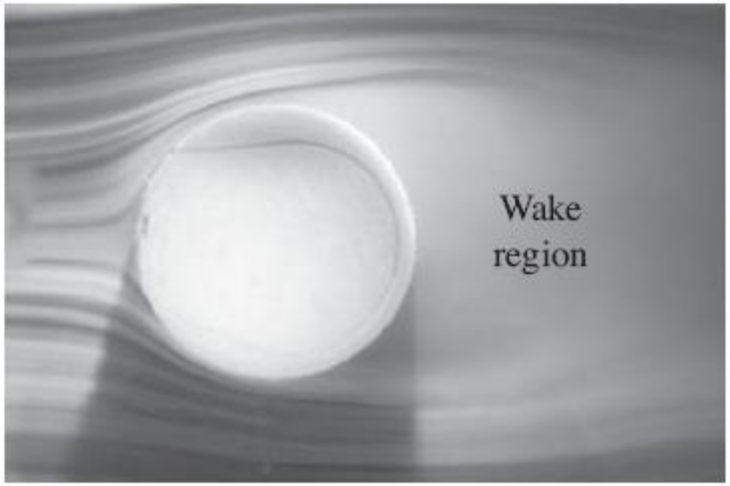


FIGURE 7–5
Separation during flow over a tennis ball and the wake region.

Wake: The region of flow trailing the body where the effects of the body on velocity are felt.

Viscous and rotational effects are the most significant in the boundary layer, the separated region, and the wake.

Heat Transfer

Local and average Nusselt numbers: $\mathrm{Nu}_x = f_1(x^*, \mathrm{Re}_x, \mathrm{Pr})$ and $\mathrm{Nu} = f_2(\mathrm{Re}_L, \mathrm{Pr})$

Average Nusselt number: $\mathrm{Nu} = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$

Average friction coefficient: $C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$

Average heat transfer coefficient: $h = \frac{1}{L}\int_0^L h_x\,dx$

Heat transfer rate: $\dot{Q} = hA_s(T_s - T_\infty)$

PARALLEL FLOW OVER FLAT PLATES

The transition from laminar to turbulent flow depends on the *surface geometry*, *surface roughness*, *upstream velocity*, *surface temperature*, and the *type of fluid*, among other things, and is best characterized by the Reynolds number.

The Reynolds number at a distance *x* from the leading edge of a flat plate is expressed as

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$

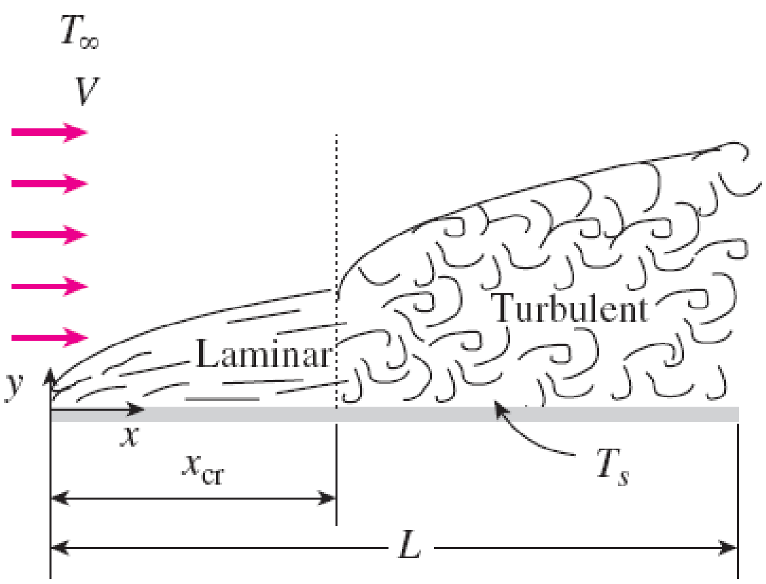


FIGURE 7–6

Laminar and turbulent regions of the boundary layer during flow over a flat plate.

A generally accepted value for the Critical Reynolds number is

$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from 10^5 to 3×10^6, depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

Friction Coefficient

$$\mathrm{Re}_x = Vx/\nu$$

Laminar: $\delta = \dfrac{4.91x}{\mathrm{Re}_x^{1/2}}$ and $C_{f,x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}}, \quad \mathrm{Re}_x < 5 \times 10^5$

Turbulent: $\delta = \dfrac{0.38x}{\mathrm{Re}_x^{1/5}}$ and $C_{f,x} = \dfrac{0.059}{\mathrm{Re}_x^{1/5}}, \quad 5 \times 10^5 \leq \mathrm{Re}_x \leq 10^7$

Laminar: $C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \leq \mathrm{Re}_L \leq 10^7$

Combined laminar + turbulent flow:

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x\ \mathrm{laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,x,\ \mathrm{turbulent}}\,dx\right)$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \leq \mathrm{Re}_L \leq 10^7$$

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$$
$$= \frac{1}{L}\int_0^L \frac{0.664}{\mathrm{Re}_x^{1/2}}\,dx$$
$$= \frac{0.664}{L}\int_0^L \left(\frac{Vx}{\nu}\right)^{-1/2} dx$$
$$= \frac{0.664}{L}\left(\frac{V}{\nu}\right)^{-1/2} \left.\frac{x^{1/2}}{\frac{1}{2}}\right|_0^L$$
$$= \frac{2 \times 0.664}{L}\left(\frac{V}{\nu L}\right)^{-1/2}$$
$$= \frac{1.328}{\mathrm{Re}_L^{1/2}}$$

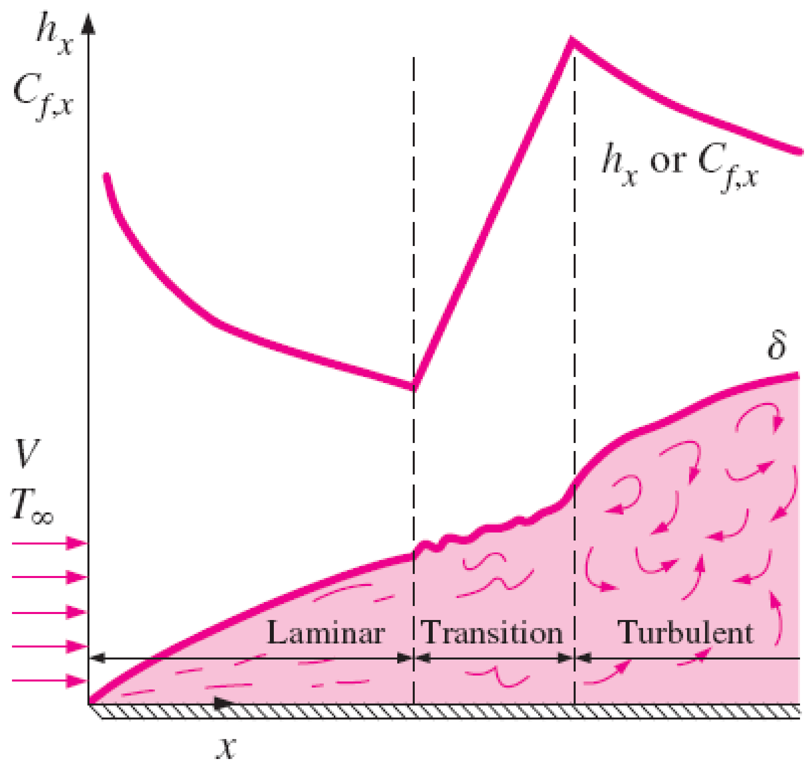
FIGURE 7–7
The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface. The values shown here are for a laminar flat plate boundary layer.

Heat Transfer Coefficient: Isothermal plate

The local Nusselt number at a location *x* for laminar flow over a flat plate may be obtained by solving the differential energy equation to be

$$\textit{Laminar:} \qquad \mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

$$\textit{Turbulent:} \qquad \mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$



These relations are for *isothermal* and *smooth* surfaces.

The local friction and heat transfer coefficients are higher in turbulent flow than they are in laminar flow.

Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction.

The variation of the local friction and heat transfer coefficients for flow over a flat plate.

Nusselt numbers for average heat transfer coefficients: Isothermal plate

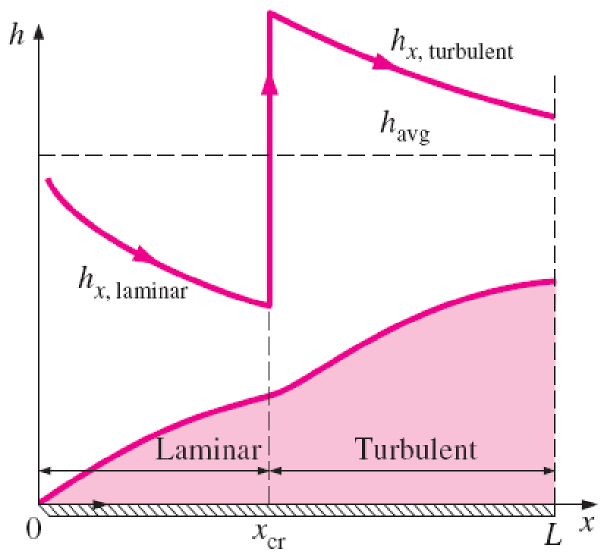
Laminar: $$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent: $$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{matrix} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{matrix}$$

Laminar + turbulent

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\,\text{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x,\,\text{turbulent}}\,dx\right)$$

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3} \qquad \begin{matrix} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{matrix}$$



Heat Transfer Coefficient: Uniform heat flux

For a flat plate subjected to *uniform heat flux*, the local Nusselt number is given by

$$\textit{Laminar:} \quad \mathrm{Nu}_x = 0.453\, \mathrm{Re}_x^{0.5}\, \mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6, \qquad \mathrm{Re}_x < 5 \times 10^5$$

$$\textit{Turbulent:} \quad \mathrm{Nu}_x = 0.0308\, \mathrm{Re}_x^{0.8}\, \mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60, \qquad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case.

When heat flux is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance *x* are determined from

$$\dot{Q} = \dot{q}_s A_s$$

$$\dot{q}_s = h_x[T_s(x) - T_\infty] \qquad \rightarrow \qquad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$

Example

EXAMPLE 7–1 **Flow of Hot Oil over a Flat Plate**

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. 7–13). Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

Example

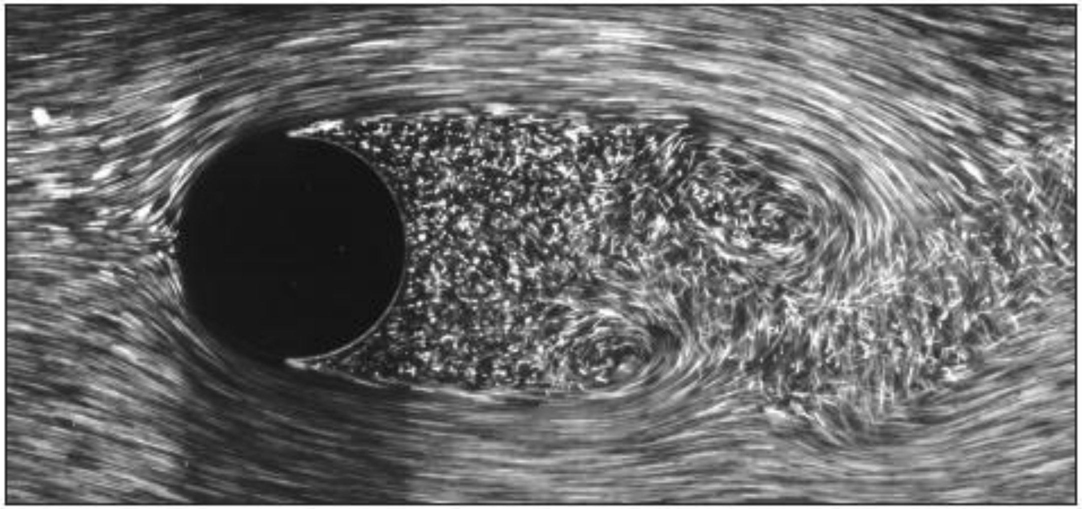
EXAMPLE 7–3 Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 1.2 m wide and 0.1 cm thick at a velocity of 9 m/min. The temperature of the plastic sheet is 95°C when it is exposed to the surrounding air, and a 0.6-m-long section of the plastic sheet is subjected to air flow at 25°C at a velocity of 3 m/s on both sides along its surfaces normal to the direction of motion of the sheet, as shown in Fig. 7–15. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\rho = 1200$ kg/m^3, $c_p = 1.7$ kJ/kg·°C, and $\varepsilon = 0.9$.

Flow Across Cylinders and Spheres

Flow across cylinders and spheres is frequently encountered in practice.
The tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes.
Many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $\text{Re} = VD/\nu$ where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\text{Re}_{cr} \cong 2 \times 10^5$. That is, the boundary layer remains laminar for $\text{Re} \lesssim 2 \times 10^5$, is "transitional" for $2 \times 10^5 \lesssim \text{Re} \lesssim 2 \times 10^6$, and becomes fully turbulent for $\text{Re} \gtrsim 2 \times 10^6$.



At very low velocities, the fluid completely wraps around the cylinder. Flow in the wake region is characterized by periodic vortex formation and low pressures.

Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder at Re = 2000.

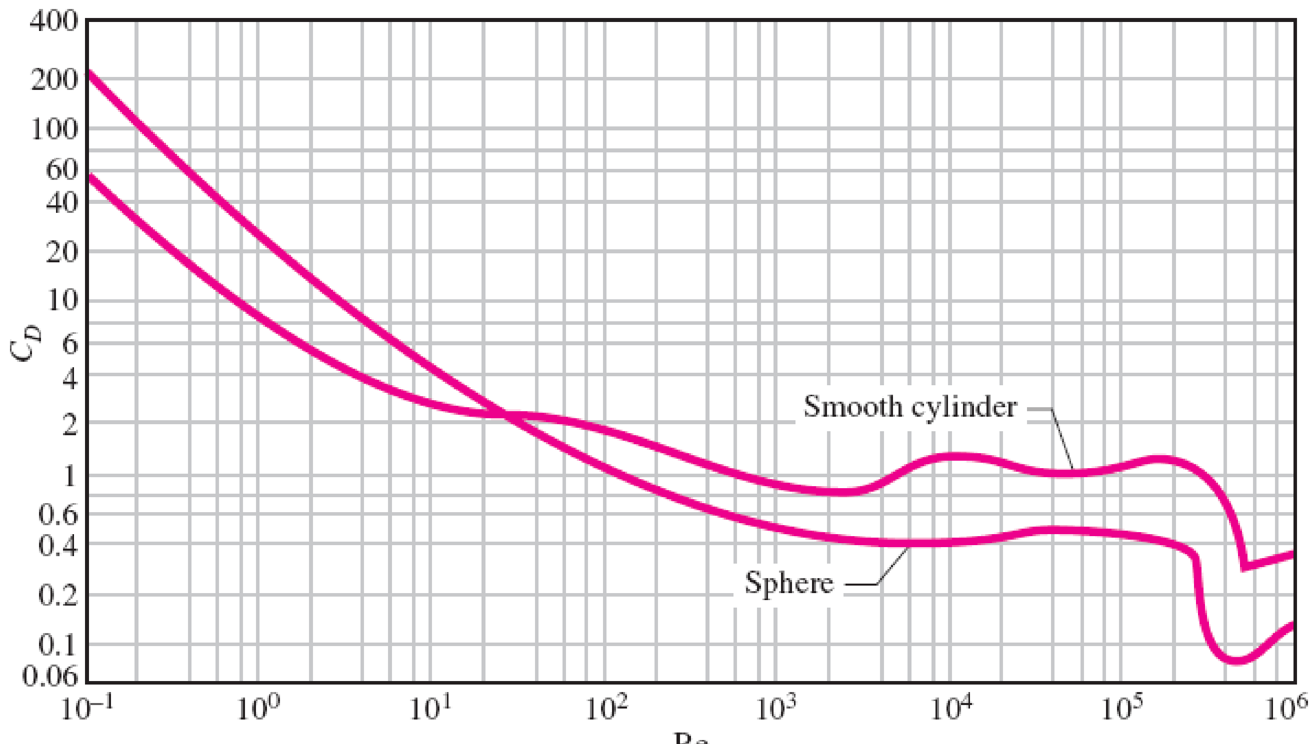
Flow Across Cylinders and Spheres

For flow across a cylinder or sphere, both the *friction drag* and the *pressure drag* can be significant.

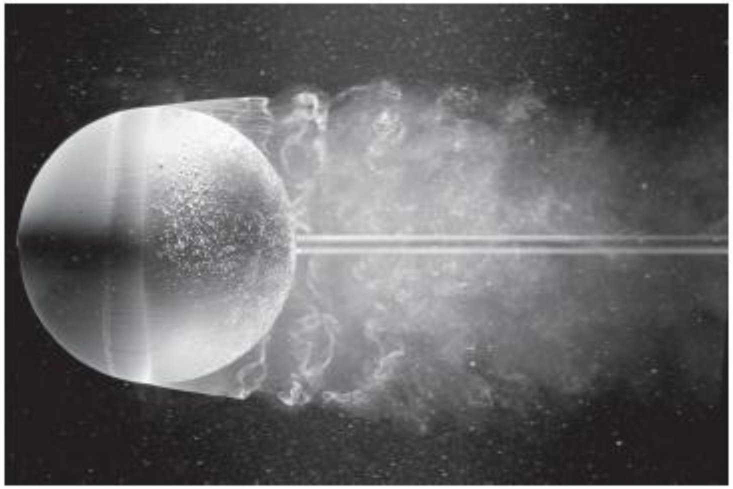
The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow.

The drag force is primarily due to friction drag at low Reynolds numbers (Re < 10) and to pressure drag at high Reynolds numbers (Re > 5000).

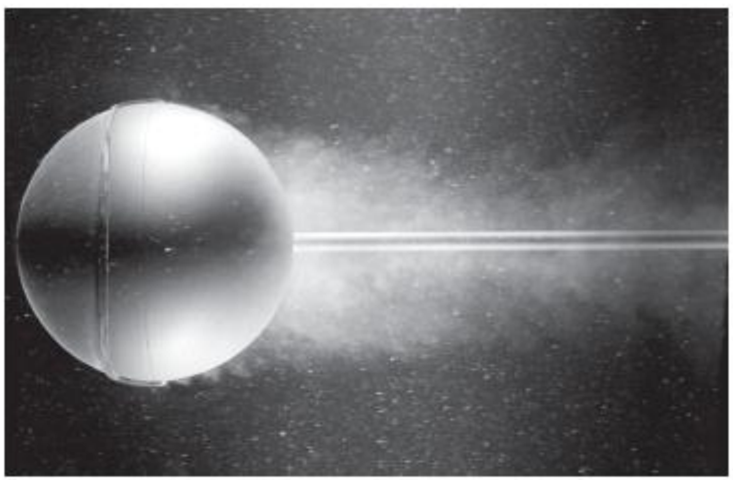
Both effects are significant at intermediate Reynolds numbers.



Average drag coefficient for cross-flow over a smooth circular cylinder and a smooth sphere.



(*a*)



(*b*)

FIGURE 7–18
Flow visualization of flow over (*a*) a smooth sphere at Re = 15,000, and (*b*) a sphere at Re = 30,000 with a trip wire. The delay of boundary layer separation is clearly seen by comparing the two photographs.

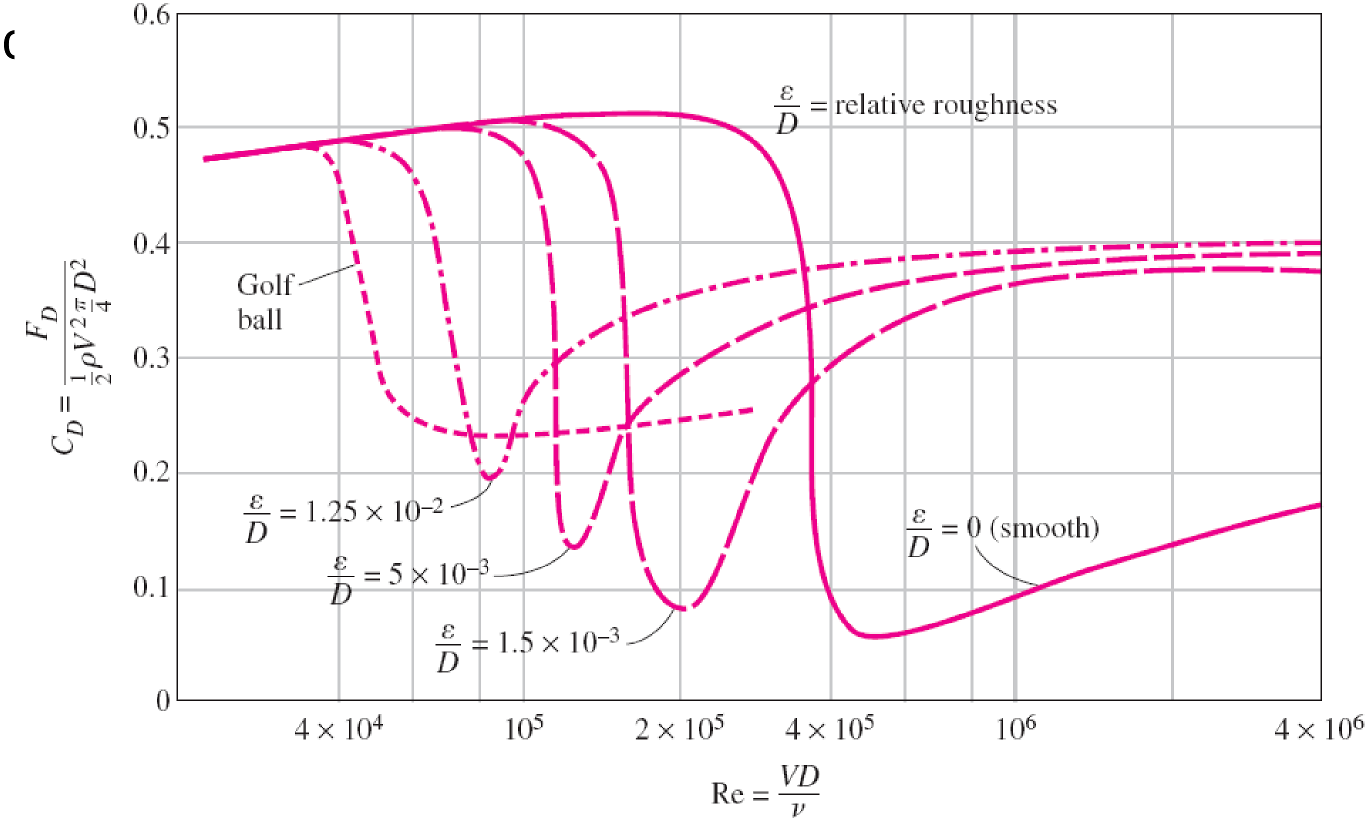
Flow separation occurs at about $\theta \cong 80°$ (measured from the front stagnation point of a cylinder) when the boundary layer is *laminar* and at about $\theta \cong 140°$ when it is *turbulent*

Effect of Surface Roughness

Surface roughness, in general, increases the drag coefficient in turbulent flow.

This is especially the case for streamlined bodies.

For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *increase* or *decrease* the drag



The effect of surface roughness on the drag coefficient of a sphere.

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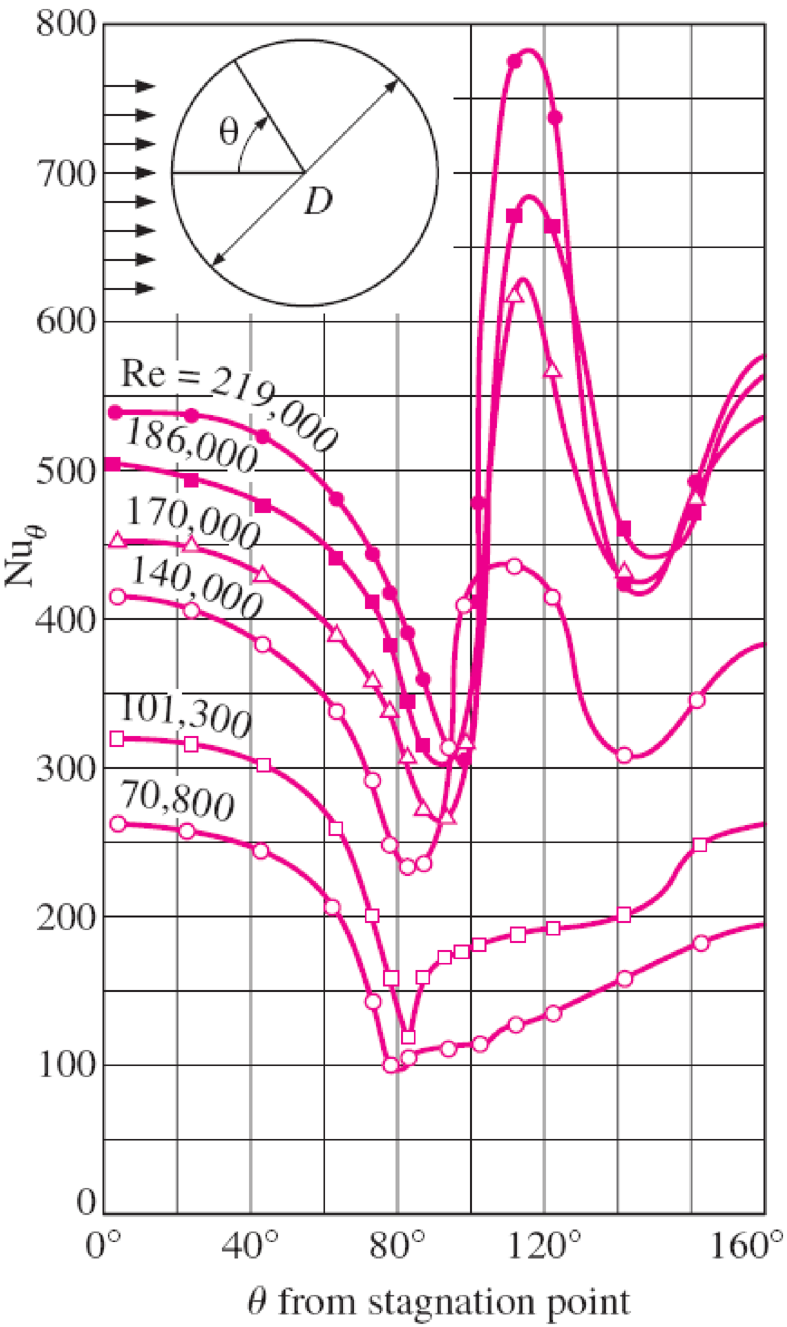
Re	C_D Smooth Surface	C_D Rough Surface, $\varepsilon/D = 0.0015$
2×10^5	0.5	0.1
10^6	0.1	0.4

FIGURE 7–20

Surface roughness may increase or decrease the drag coefficient of a spherical object, depending on the value of the Reynolds number.

Heat Transfer Coefficient

- Flows across cylinders and spheres, in general, involve *flow separation*, which is difficult to handle analytically.
- Flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.



Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross flow of air.

For flow over a *cylinder*

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$\mathrm{RePr} > 0.2$

The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$

For flow over a *sphere*

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$3.5 \leq \mathrm{Re} \leq 80{,}000$ and $0.7 \leq \mathrm{Pr} \leq 380$

The fluid properties are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s.

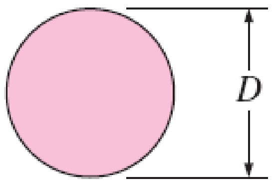
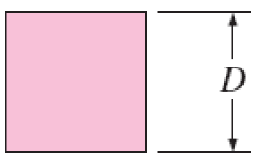
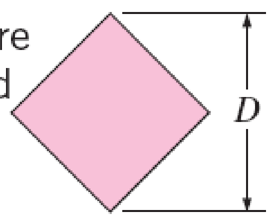
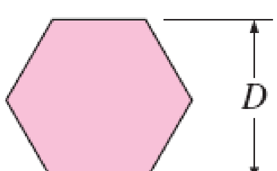
$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$n = \frac{1}{3}$

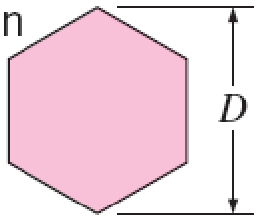
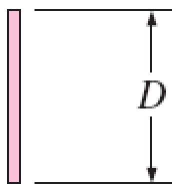
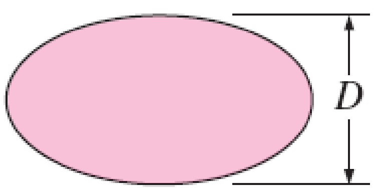
Constants *C* and *m* are given in the table.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. They are applicable to

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

Summary

- Drag and Heat Transfer in External Flow
  - ✓ Friction and Pressure Drag
  - ✓ Heat Transfer
- Parallel Flow Over Flat Plates
  - ✓ Friction Coefficient
  - ✓ Heat Transfer Coefficient
  - ✓ Flat Plate with Unheated Starting Length
  - ✓ Uniform Heat Flux
- Flow Across Cylinders and Spheres
  - ✓ Effect of Surface Roughness
  - ✓ Heat Transfer Coefficient